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EFFECT OF SOLID-PARTICLE OR DROPLET ADMIXTURE ON THE STRUCTURE OF A TURBULENT GAS JET

G. N. ABRAMOVICH
Moscow Aircraft Engineering Institute, Moscow, U.S.S.R.

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Аннотация—В предлагаемой приближенной теории считается, что в процессе турбулентного пульсационного движения элементарный газовый обьем увлекает населяющие его дискретные частицы и тормозится силой суммарного лобового сопротивления последних, в связи с чем пульсационные скорости уменьшаются.

Показано, что мелкие частицы сильнее подавляют турбулентность, чем крупные. Получены простые зависимости, характеризующие влияние концентрации и размеров частиц примеси на степень турбулентности, напряжение трения, толщину струи и изменение скорости и концентрации по оси струи, которые удовлетворительно согласуются с опытными данными.

NOMENCLATURE

x,	abscissa (distance from jet beginning);
y,	ordinate (distance from jet axis);
δ',	characteristic jet thickness;
δ'_u,	thickness defined from velocity field;
$\delta_\varkappa$,	thickness defined from concentration field;
l,	characteristic dimension of turbulent fluid element;
l_T,	turbulent mixing length;
u, v,	averaged gas velocity components:
u_m,	velocity at jet axis;
u_c,	mean characteristic velocity in jet cross-section;
u_{co},	as above but without addition:
u', V'	fluctuating velocity components;
u'_0, V'_0,	as above, but at the beginning of mixing length;
u'_k, V'_k,	as above. at the end of mixing length;
V_p,	velocity of particle due to fluctuating gas motion;
V_{p0}, V_{pk},	as above, at the beginning and at the end of mixing length;
$V_\sim = V' - V_p$,	relative velocity of gas and particles in fluctuating motion;
$V_{\sim k}$,	as above, towards the end of fluid element motion;
ρ,	mixture density;
ρ_g.	gas density;
ρ_p,	particle density;
G_g,	mass flow rate of gas;
G_p,	mass flow rate of mixture;
$\varkappa = (G_p/G_g)$,	relative weight (concentration) of mixture;
$\varkappa_m$,	as above at jet axis;
$\varkappa_1$,	characteristic values of concentration in jet cross section;
V_{pa}, V_{pb} . . . ,	velocities of particles with diameters D_a, D_b, . . .
$\varkappa_a$, $\varkappa_b$,	concentrations of particles with diameters D_a, D_b, . . .
P,	pressure;
p',	pressure fluctuation;

P'_{max},	maximum characteristic value of pressure fluctuation over jet cross-section;
$\eta = y/\delta_u$,	dimensionless coordinate;
η_i,	dimensionless ordinate of point of maximum pressure fluctuation;
$\eta_p = (\delta_\varkappa/\delta_u)$,	ratio of diffusion and dynamic mixing zone thicknesses;
m,	mass of added particle;
C_x,	drag coefficient of particle in fluctuating gas motion;
μ_g,	dynamic viscosity of gas.

THE PROBLEM of non-uniform turbulent propagation is of great practical interest and is therefore an object of recent researches both in the U.S.S.R. and other countries.

The most complicated problem is that concerning the effect of discrete addition of solids or liquid drops to a gas jet.

Let us introduce the concept of mean density of a jet at a given point.

$$\rho = \rho_g(1 + \varkappa) \tag{1}$$

where ρ_g is the gas density and $G_p/G_g = \varkappa$ is the weight concentration of the addition. Experiments have shown that a jet formed by mixing two gases of different molecular weights behaves in a different way than a jet containing admixture of discrete particles, with the mean densities of the jets being equal. In the latter case, the jet appears to be narrower and to penetrate deeper. This may probably be attributed to the change of the turbulent jet structure due to the effect of the solid particles. Within the first approximation, we shall show that the effect of solid particles on fluctuating velocities and averaged parameters of a turbulent flow can be evaluated on the basis of the Prandtl mixing-length theory.

Generally, the size D of solid particles or drops entrained by a gas jet is several orders smaller than the size l of the turbulent fluid element whose behaviour determines the structure of the turbulent flow.

In the process of turbulent fluctuating motion, discrete gas volume (fluid element) entrains the foreign particles populating this gas volume and is retarded under the action of total drag force which results in a decrease of fluctuating velocity components of a turbulent flow

Let us now examine an important practical case when the weight of addition per unit volume is considerable while its volume fraction is relatively small. In this case, the aerodynamic drag of all heavy particles around which a gas particle flows is relatively great and is equal to the sum of drags arising when each particle is considered to be an isolated body.

Let us consider the effect of heavy particles on the magnitude of transversal fluctuating velocity (V') of a turbulent jet, determining as is well known the intensity of jet thickening [1].

When a turbulent fluid element is developed, its fluctuating velocity is proportional to the transversal gradient of the mean flow velocity,

$$|V'_0| \sim l_T \frac{du}{dy} \tag{2}$$

where l_T is the mean turbulent mixing-length.

In the existing semi-empirical turbulence theories the fluctuating velocity of the fluid element is considered to be retained during the course of its entire "life", i.e. from the instant of isolation from one layer till confluence with the other layer; the loss of fluid element individuality occurs abruptly and thus causes fluctuations in velocity (as well as in pressure, temperature, concentration, etc).

From the law of momentum conservation it follows that in the presence of heavy foreign particles, the velocity V' of the fluid element decreases proportionally to the increase in the velocity V_p of particles entrained by the fluid element while the proportionality factor is equal to the ratio of masses of the admixture and gas, i.e. is equal to the weight concentration of the admixture:

$$V'_0 - V' = \varkappa(V_p - V_{p0}) \tag{3}$$

where V_{p0} is the initial velocity of heavy particles. This radiation is true for a monodispersed and homogeneous admixture. In case of a polydispersed admixture, the right hand side of equation (3) becomes complicated, but we shall treat this case later.

At the instant when a turbulent fluid element is formed which begins moving with the initial velocity V'_0, heavy particles entrained by the previous element enter into it and have with equal probability the velocity V_{p0} in the same or opposite direction. Therefore, the mean value of the initial velocity of heavy particles can be assumed to be zero ($V_{p0} \approx 0$).

This simplifies equation (3) to the form

$$V'_0 - V' = \varkappa V_p. \tag{4}$$

As follows from equation (4), in the case of a large weight concentration of the addition ($\varkappa \gg 1$), the heavy particles move with a relatively small velocity ($V_p \ll V'_0$) even though the fluid element is considerably retarded ($V'_0 \gg V'$).

The subsequent solution of this problem depends on the rate at which the final velocities of the fluid element and of the heavy particles approach each other.

If the size and velocity of particles are small, the "inter-element" flow of gas around the particle can be laminar, and the aerodynamic drag coefficient is relatively high. When the mass of each particle is relatively small, its velocity becomes rapidly equal to the gas velocity ($V_{pk} \approx V'_k$). Therefore according to equation (4), the final velocity of the fluid element is

$$V'_k = V'_0 \frac{1}{1+\varkappa}. \tag{5}$$

Hence the ratio of turbulent shear stress in a flow with admixture to that in a pure gas ($u' \sim V'$) is

$$\frac{\tau}{\tau_0} = \frac{\rho \overline{u'_k V'_k}}{\rho_g \overline{u'_0 V'_0}} = \frac{1}{1+\varkappa}.$$

As is known, the rate of jet thickening along its length ($d\delta/dx$) is proportional to the tranverse fluctuating velocity [1].

In the present case

$$\frac{d\delta}{dx} \sim \frac{v'_k}{u_c} \text{ or } \frac{d\delta}{dx} \sim \frac{v'_k}{v'_0}\frac{v'_0}{u_m}\frac{u_m}{u_c} \tag{6}$$

where u_c is the characteristic value of a mean flow velocity, u_m is the velocity along the jet axis and the subscript "0" indicates the absence of admixture, whereas in a submerged jet,

$$V'_0 \sim u_m, \frac{V'_0}{u_{c0}} \sim \frac{d\delta}{dx}_0 = c = 0{\cdot}22.$$

In a jet of a variable density (with admixture) using the suggestion given in reference [1], we can write

$$u_c = \frac{\int_0^\delta \rho u \, dy}{\int_0^\delta \rho \, dy}. \tag{7}$$

Numerous experiments show that in a jet containing both gaseous and liquid droplets or dispersed solid additions the relative velocity profile remains unchanged in the cross-section, and in the main jet region it can be expressed by Schlichting's formula

$$\frac{u}{u_m} = \left[1 - \left(\frac{y}{\delta_u}\right)^{1.5}\right]^2 \tag{8}$$

where u is the velocity at a distance y from the jet axis, δ_u is the distance between the axis and the dynamic jet boundary (jet radius).

The concentration distributions are satisfactorily described by the same relation, but the corresponding thickness of "addition zone" usually differs from that of the "dynamic zone" ($\delta_\varkappa \neq \delta_u$):

$$\frac{\varkappa}{\varkappa_m} = [1 - \eta^{1{\cdot}5}\eta_p^{-1{\cdot}5}]^2 \tag{9}$$

where $\eta = (y/\delta_u)$, $\eta_p = \delta_\varkappa/\delta_u$, $\eta\eta_p^{-1} = y/\delta_\varkappa$. $\varkappa_m$ is the concentration at the jet axis. According to some experimental results, we can express the relation $\eta_p(\varkappa_m)$ by the curve shown in Fig. 1*. In the range $0{\cdot}5 \leqslant \varkappa_m \leqslant 10$, we have $\eta_p < 1$, i.e. the velocity profile is wider than the concentration profile. With no addition, $(\varkappa = 0)$, from equations (7) and (8), we get $u_{co} = 0{\cdot}45\, u_m$.

In the presence of a finely dispersed addition $(V_{pk} \approx V_k')$

$$\frac{\mathrm{d}\delta}{\mathrm{d}x} = \frac{0{\cdot}45c}{1 + \varkappa_1} \frac{u_m}{u_c} \tag{10}$$

where $\varkappa_1$ is some characteristic value of concentration in the jet cross-section.

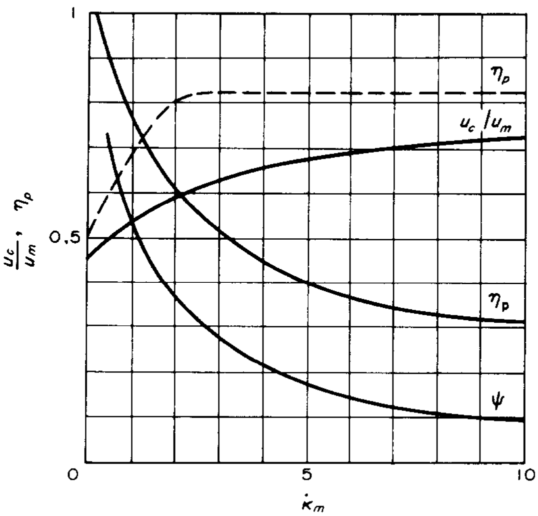

FIG. 1.

Using equations (1) and (7) we obtain

$$\frac{u_c}{u_m} = \frac{\int_0^{\eta_p} (u/u_m)\mathrm{d}\eta + \varkappa_m \int_0^{\eta_p} (\varkappa/\varkappa_m)/(u/u_m)\mathrm{d}\eta}{1 + \varkappa_m \int_0^{\eta_p} (\varkappa/\varkappa_m)\,\mathrm{d}\eta}$$

when $\eta_p < 1$.

Hence, according to equations (8) and (9) we have

$$\frac{u_c}{u_m} = \frac{0{\cdot}45 + \varkappa_m(0{\cdot}45\eta_p - 0{\cdot}164\eta_p{}^{2\cdot5} + 0{\cdot}029\eta_p{}^4)}{1 + 0{\cdot}45\varkappa_m\eta_p}. \tag{11}$$

For determination of the local value of the angular coefficient of jet-thickening, we are to find, using equation (10), a method for selecting the corresponding value of characteristic concentration $\varkappa_1$ in the section.

We assume that the zone of maximum turbulent pressure fluctuations is the dominant one when turbulence is being generated in the section:

$$p'_{\max} = \rho \overline{V}{}'^2_{\max} \sim [(1 + \varkappa)(\mathrm{d}u/\mathrm{d}y)^2]_{\max}$$

Therefore, we shall consider the value $\varkappa = \varkappa_1$ to be characteristic at a point where the following quantity assumes a maximum value

$$\varphi = \varphi_{\max} = [1 + \varkappa_m(1 - \eta^{1\cdot5}\eta_p^{-1\cdot5})^2](\eta^{0\cdot5} - \eta^2)^2$$

according to equations (8) and (9).

By solving the problem for the maximum value, we find the coordinates of the point η_1 from the equation

$$\varkappa_m = \frac{1 - 4x}{3bx(1 - x)(1 - bx) - (1 - 4x)(1 - bx)^2} \tag{12}$$

where $x = \eta_1^{1\cdot5}$, $b = \eta_p^{-1\cdot5}$.

The value of $b(\varkappa_m)$ is found from the curve $\eta_p(\varkappa_m)$ (Fig. 1) and then $x(\varkappa_m)$ is determined. From equation (9) we find the characteristic value of concentration

$$(\varkappa_1/\varkappa_m) = (1 - \eta_1^{1\cdot5}\eta_p^{-1\cdot5})^2 = (1 - bx)^2. \tag{13}$$

The calculations show that the quantity $(\varkappa_1/\varkappa_m)$ varies very little over the range $0{\cdot}5 \leqslant \varkappa_m \leqslant 10$,

$$(\varkappa_1/\varkappa_m) = 0{\cdot}76 \pm 0{\cdot}03. \tag{14}$$

The following curve also shown in Fig. 1 is calculated with the help of equations (10), (11) and (13)

$$\frac{1}{C}\frac{\mathrm{d}\delta}{\mathrm{d}x} = \Psi(\varkappa_m)$$

* The solid curve $\eta_p(x_m)$ In Fig. 1 is obtained from the experiments on mixing of two gases of different molecular weights (Freon and air). This curve is probably valid for the case of very finely dispersed admixtures ($\eta < 30\mu$). The dotted curve in Fig. 1 corresponds to the experimental data on large added particles ($\eta = 30$–120μ).

where Ψ is the ratio of the thickness of a jet with àddition to that of a "pure" jet.

The results obtained are valid only for an addition of relatively finely dispersed particles which are totally entrained by the turbulent fluid elements. Estimations show that with a mean jet velocity in the jet of an order of 50 m/s, solid particles of 10–30 μ dia. are totally entrained. For larger particles, the problem becomes more complicated. These particles "have no time" to acquire the velocity of the fluid element during its motion ($V_{pk} < V'_k$).

Let us introduce the concept of relative velocity

$$V_{\sim} = V' - V_p.$$

Then from equation (4) we get

$$\frac{V'_k}{V'_0} = \frac{1 + \varkappa_1 (V_{\sim k}/V'_0)}{1 + \varkappa_1}. \qquad (15)$$

Thus, if the velocity of solid particles is less than that of the fluid element ($V_{\sim k} = V'_k - V_{pk} > 0$), we substitute expression (15) for (5) in formula (6); the intensity of jet thickening in this case proves to be greater than that in the previous case ($V_{\sim k} = 0$)

$$\frac{d\delta}{dx} = 0{\cdot}45c\frac{u_m}{u_c}\frac{1 + \varkappa_1 (V_{\sim k}/V'_0)}{1 + \varkappa_1}. \qquad (16)$$

The relation between the current value of the element velocity V' and that of a solid particle (sphere) is given by the second law of mechanics

$$m_p\frac{dV_p}{dt} = C_x\frac{\mu D^2}{4}\frac{\rho_g}{2}(V' - V_p)^2.$$

But the mass of a particle is

$$m_p = \frac{4}{3}\mu\rho_p\left(\frac{D}{2}\right)^3.$$

Hence

$$\frac{dV_p}{dt} = \frac{3}{4}\frac{C_x}{D}\frac{\rho_g}{\rho_p}(V' - V_p)^2.$$

The path travelled by a particle in time dt is equal to $dy = V_p dt$, is therefore

$$\frac{V_p dV_p}{(V' - V_p)^2} = \frac{0{\cdot}75C_x}{D}\frac{\rho_g}{\rho_p}dy. \qquad (17)$$

An estimation of the effect of unsteady flow on the drag coefficient, which was made using the data of [2], has shown that this effect does not exceed 10 per cent. We can therefore make use of the ordinary relations $C_x(R_D)$ for a steady flow by determining the Reynolds number based on the relative velocity of fluctuating motion

$$R_D = \frac{\rho_g(V' - V_p)D}{\mu_g}$$

where μ_g is the dynamic viscosity of the gas.

In case of laminar flow we can use Stokes law which gives exact values when $R_D \leqslant 1$; at $R_D \approx 10$ it is accurate within 30 per cent

$$C_x = \frac{24}{R_D}.$$

With the boundary condition $V_{\sim} = V'_0$ we get from equation (17) for $y = 0$

$$f\left(\frac{V_{\sim}}{V'_0}\right) = \ln\frac{V'_0}{V_{\sim}} + \frac{V_{\sim}}{V'_0} - 1 = \frac{18}{R_M}\frac{\rho_g}{\rho_p}\left(\frac{l_T}{D}\right)^2\frac{y}{l_T}(1 + \varkappa_1)^2. \qquad (18)$$

Here the Reynolds number for the fluid element is introduced

$$R_M = \frac{\rho_g V'_0 l_T}{\mu_g}$$

Over the range $R_D = 5 \times 10^2 - 2 \times 10^5$, the experimental curve of $C_x(R_D)$ is steep and we can take in this region $C_x \approx$ const $\approx 0{\cdot}5$. For $R_D > 2 \times 10^5$ (supercritical flow around a sphere) we can assume that $C_x \approx$ const $\approx 0{\cdot}2$. For $C_x =$ const. ($V = V'$ for $y = 0$) we obtain from equation (17)

$$\varphi\left(\frac{V_\sim}{V'_0}\right) = \ln\frac{V_\sim}{V'_0} + \frac{V'_0}{V_\sim} - 1 = 0{\cdot}75C_x(1 + \varkappa_1)^2\frac{l_T}{D}\frac{y}{l_T}. \qquad (19)$$

When $y = 1_T$ the dimensionless relative velocity of the fluid element at the end of unit displacement $(V'_{\sim k}/V'_0)$ is determined from equations (18) or (19) and then substituted into equation (16). These relations characterizing a jet with relatively large particles are valid only for a fine dispersed addition.

For a polydispersed addition we have the following relation:

$$\varkappa = \varkappa_a + \varkappa_b + \ldots \qquad (20)$$

where $\varkappa_a$, $\varkappa_b$.... are the weight concentrations of particles of diameters D_a,D_b...

Let the final velocities of accelerated particles of different diameters be V_{pa},V_{po}...

Then equation (4) takes the following form for a polydispersed mixture

$$V'_0 - V' = \varkappa_a V_{pa} + \varkappa_b V_{pb} + \ldots \qquad (21)$$

Further calculations are made by simultaneously solving the system of equations (17) (written for particles of every size) and equation (21).

Thus, when $y = 1_T$, we can determine the values of V_{pa}, V_{pb}... and the magnitude of V'_k which is to be substituted into equation (16) and gives db/dx for a polydispersed mixture.

Other jet parameters (concentration and velocity fields, changes in jet thickness along the length of the jet, etc.) are found from the momentum and mass conservation laws as applied to the addition.

The calculations become somewhat more complicated when we take into account the inequality of local averaged velocities of gas and solids. If these velocities are different, then due to rotation of solid particles (caused by flow swirling) an aerodynamic lift force appears displacing particles in the direction normal to the jet axis. This should change the shape of the curve $\eta_p(\varkappa_m)$. Although the available experimental results do not contradict the simplified theory proposed here, yet a thorough and comprehensive experimental study is required for a more strict evaluation.

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Abstract—In the approximate theory presented here it is assumed that in the process of turbulent fluctuating motion, a fluid element entrains discrete particles populating this element and is retarded by a total drag force which results in a decrease of the fluctuating velocity.

It is shown that the decreasing effect of fine particles on the turbulence level is stronger compared to that of coarse particles. Simple relations are obtained which describe the effect of concentration and particle dimensions on the turbulence level, shear stress and concentration distribution along the jet axis. The relations obtained show good agreement with the available experimental data.

EFFET DU MÉLANGE DE PARTICULES SOLIDES OU DE GOUTTES SUR LA STRUCTURE D'UN JET TURBULENT GAZEUX.

Résumé—Dans la théorie approchée présentée, on suppose que dans le processus de mouvement fluctuant turbulent, un élément de fluide entraine des particules discrètes peuplant cet élément et est retardé par une force totale de trainée ce qui s'accompagne d'une diminution de vitesse fluctuante. On montre que l'effet de décroissance des particules fines sur le niveau de turbulence est plus fort, comparé à celui des grosses particules. On obtient des relations simples qui décrivent l'effet de la concentration et des dimensions des particules sur le niveau de turbulence, la contrainte tangentielle et la distribution de concentration le long de l'axe du jet. Les relations obtenues sont en bon accord avec les résultats expérimentaux valables.

DER EINFLUSS DER BEIMISCHUNG VON FESTKÖRPERCHEN ODER TROPFEN AUF DIE STRUKTUR EINES TURBULENTEN GASSTROMES

In der hier vorgestellten Näherungstheorie wird angenommen, dass im Verlauf der turbulenten Austauschbewegung ein Fluidelement diskrete Partikel aufnimmt und durch die gesamte Widerstandskraft verzögert wird. Die Austauschgeschwindigkeit wird dadurch verringert. Es wird gezeigt, dass diese vermindernde Wirkung auf den Turbulanzgrad bei kleinen Partikeln stärker ist als bei grossen Teilchen. Es werden einfache Beziehungen aufgestellt, die den Einfluss der Konzentration und der Grösse der Teilchen auf den Turbulenzgrad, dic Schubspannung und die Konzentrationsverteilung entlang der Strömungsachse beschreiben.

Die aufgestellten Bezeichnungen zeigen gute Übereinstimmung mit vorhandenen experimentellen Werten.